Linear Waves Primer

13.012 Pre-Lab Handout

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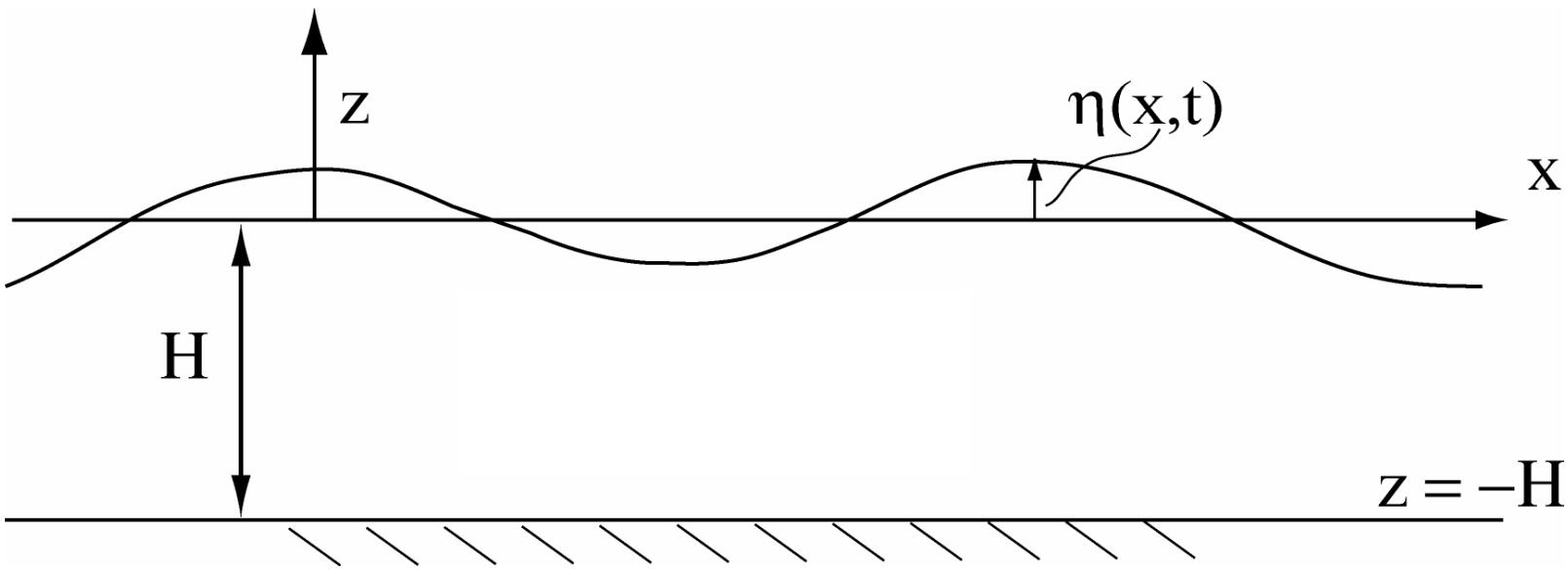
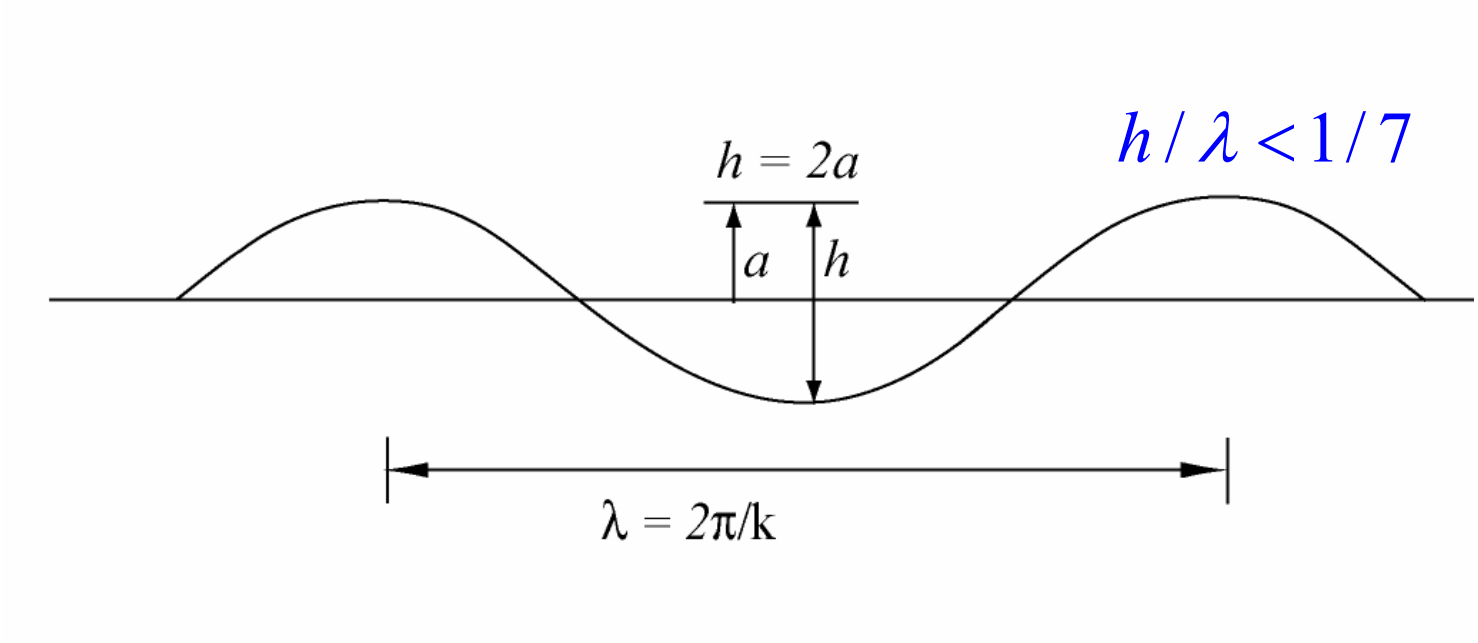
Plane Progressive Waves

Linear free-surface gravity waves can be characterized by their amplitude, a, wavelength, $\lambda = 2\pi/k$, and frequency, ω.

$$\eta(x,t) = a\cos(kx - \omega t)$$



Linear Waves

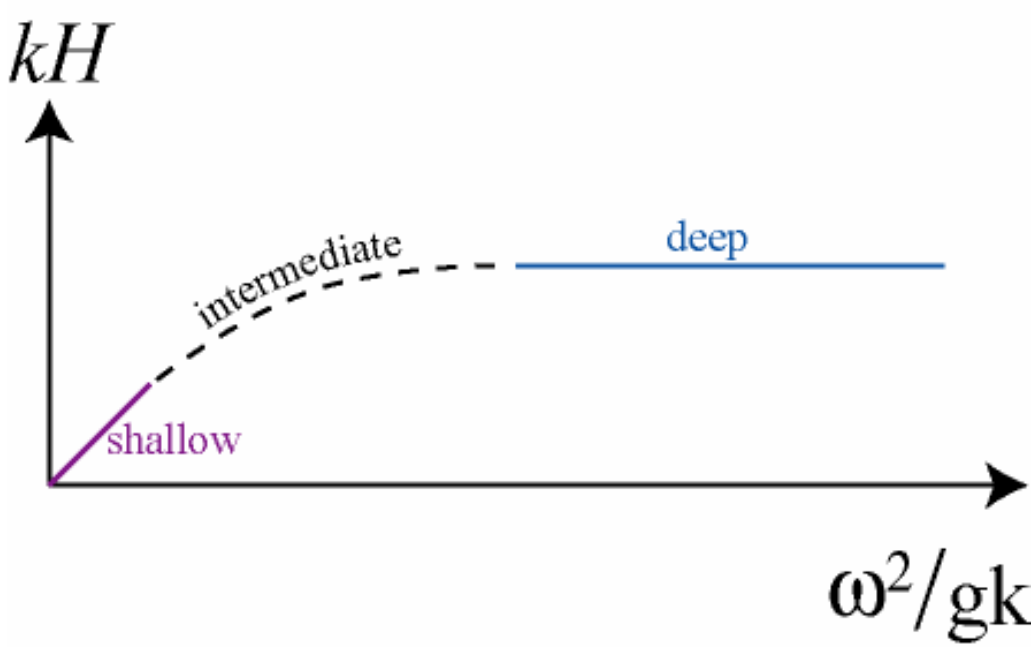


- *a* is wave amplitude, *h* = 2*a*
- λ is wavelength, $\lambda = 2\pi/k$ where k is wave number
- Waves will start to be non-linear (and then break) when $h/\lambda > 1/7$

Dispersion Relationship

$$\omega^2 = gk\tanh(kH)$$

- <u>Approximations</u>
  - As $kH \to 0$ $\tanh(kH) \to kH$ $\therefore \omega^2 \cong gk^2 H$ (shallow)
  - As $kH \to \infty$ $\tanh(kH) \to 1$ $\therefore \omega^2 \cong gk$ (deep)



Phase and Group Speed

$$V_p = \frac{\omega}{k}$$

Phase Speed = speed of the wave crest

$$V_g = \frac{d\omega}{dk}$$

Group Speed = speed of the wave energy

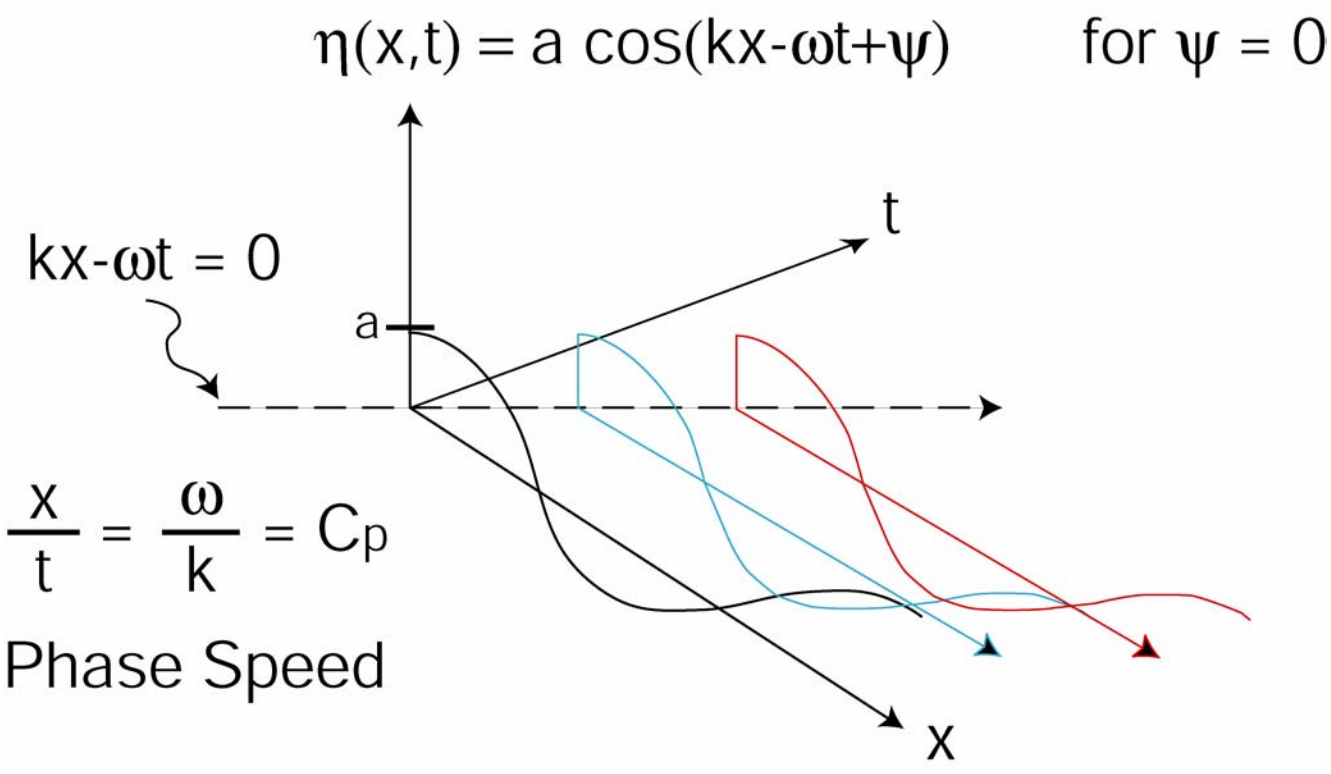
<u>Shallow water</u>

$$V_g = V_p$$

<u>Deep water</u>

$$V_g = \frac{1}{2} V_p$$

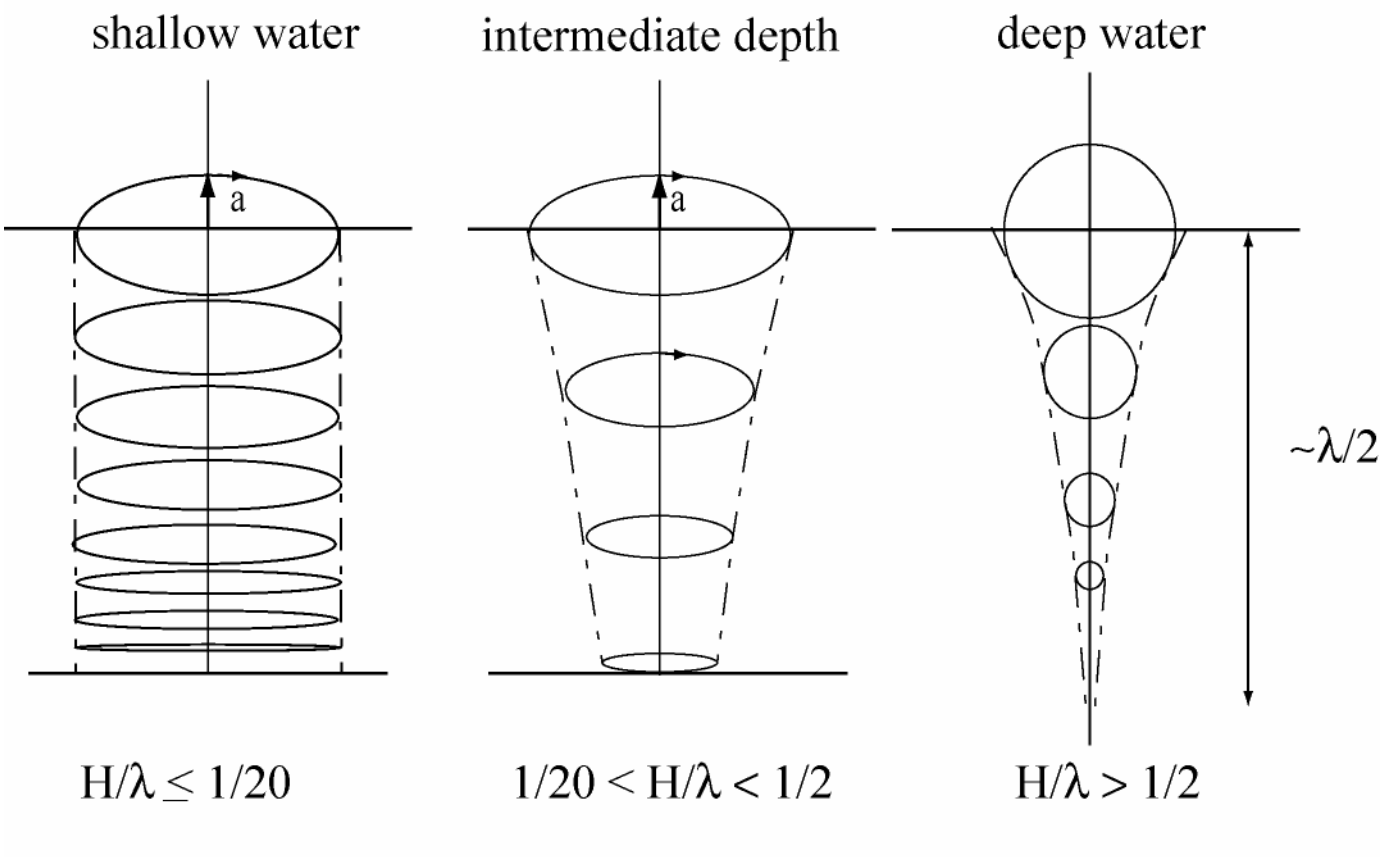
Phase Speed



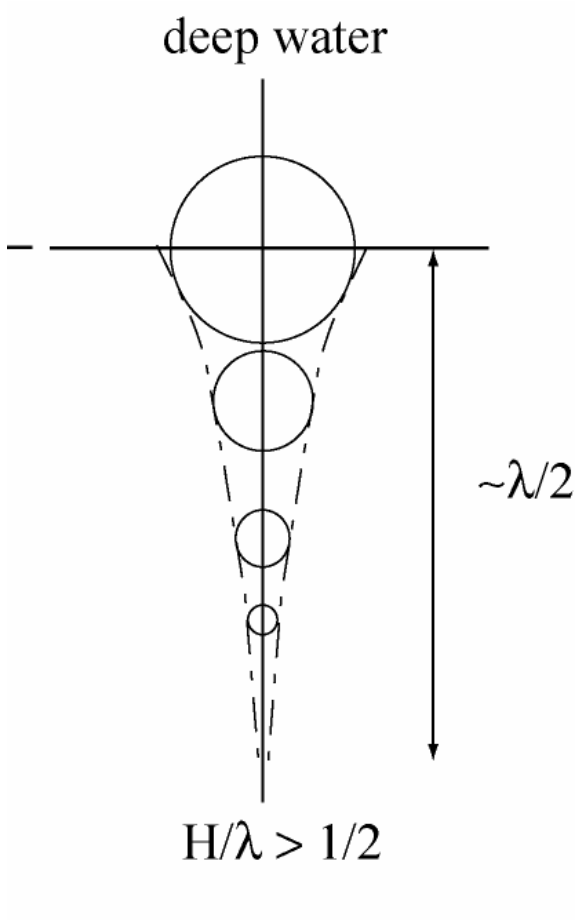
Velocity at which a wave crest is moving

Particle Orbits

Under the waves particles follow distinct orbits depending on whether the water is shallow, intermediate or deep. Water is considered deep when water depth is greater than one-half the wavelength of the wave.



Particle Orbits in Deep Water



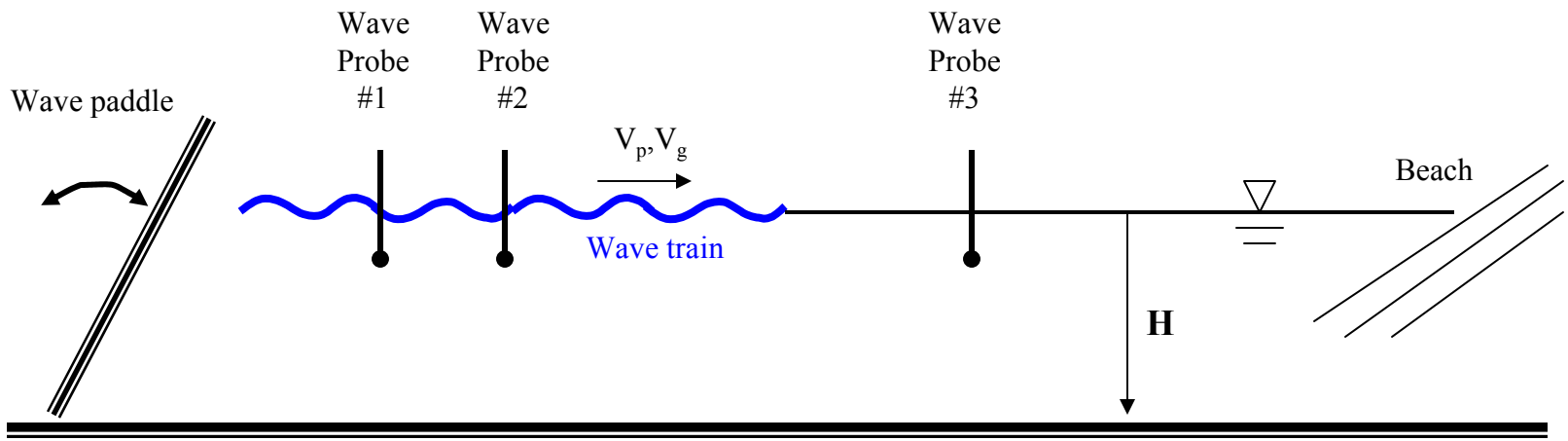
$$H \to \infty \quad (kH \gg 1)$$

$\omega^2 = gk \Leftrightarrow$ dispersion relationship

Circular orbits with
<u>exponentially</u>
decreasing radius

Particle motion extinct at $z \cong -\lambda/2$

Lab Setup & Procedure



1. Input wave frequency and amplitude to wave maker using potentiometers on the Towtank bridge.
2. Use data acquisition to record wave probes measurements of wave amplitude as a function of time for multiple frequencies and amplitudes.
3. Use a stop watch to determine a rough estimate of phase and group speed.
4. Observe surface contraction and dilation with pepper/glitter.
5. Observe particle orbits below surface with pepper, if possible.